Detecting and characterising exoplanets
Teachers' notes

Maths curriculum topics involved in this task are:

- Interpreting graphs
- Scale factor and area factor
- Ratio & proportion
- Standard form
- Circles & circle formulae
- Application of trig graphs
- Volume of a sphere

1. The time scale is given on the axis running along the bottom of the plot. The duration of the transit event – the difference between the start time and the end time – can be read off this scale; about 7 hours. See if you can get your students to explain the shape of the dip in the lightcurve. Why is it not strict, straight-sided profile, like a rectangular dip in the lightcurve? Think about the circular outline of the planetary disk as it first crosses into the stellar disk, and then out again at the end of the transit. What other property might a planet have that would affect how it blocks starlight – an atmosphere.

2. The light curve in Fig.2 dips down to around 97.5%, meaning a depth of transit of around 2.5%.

3. The drop in starlight brightness is determined by how much of the star's disk is blocked out by the disk of the exoplanet. Clearly, a large planet will block more starlight, and so cause a deeper transit event, than a small planet. Specifically, the decrease in light intensity is equal to the ratio of the area of the dark disk of the planet to the area of the bright star's disk. And if you know the area of a circular disk, you can simply calculate its radius. Using other techniques, astronomers are able to determine the radius of the host star, and so the transit method allows you to calculate the radius of the exoplanet.

4. The depth of the transit event is 2.5% (Q2), meaning that the ratio of the area of the disk of the planet to the area of the disk of the star is 0.025. The area of a circle is proportional to its radius squared, and so the ratio of the planetary radius to the stellar radius is $\sqrt{0.025} = 0.158$ (to 3 d.p.). Thus, the radius of the planet is $0.158 \times 5 \times 10^8$ m $= 7.91 \times 10^7$ m.

5. $7.91 \times 10^7 / 7.0 \times 10^7 = 1.13$, meaning that the exoplanet has a radius (or diameter) 13% greater than Jupiter.

6. The time interval between separate transits from the same planet tells you the orbital period of that planet – how long its year is. Looking at Fig.3, the interval for this exoplanet is about 4.5 Earth days. Get your students to comment on this figure – it's an exceedingly short year! Using Kepler's laws of orbital motion, we can use this orbital period to calculate the distance the planet orbits its star, and thus what temperatures the planet is likely to be. Why might it be that the first exoplanets we discovered all had very short years? Several reasons: planets orbiting closely to their star block-out a lot of light and so are easier to detect by

the transit method, and in order to be confident that you have discovered a new planet, and are able to calculate its orbital period, you need to witness several transits, and so obviously the closer the orbit the more quickly you see several transits.

7. An exomoon – a moon orbiting a planet orbiting another star! A large moon orbiting a planet will sometimes be pulling it forward in its orbit (so that it might transit its star from our point of view earlier) and sometimes pulling it back (so that it might transit later). This technique, called Transit Timing Variation (TTV) has been proposed as a way to hunt for exomoons, but as yet, none have been discovered.

8. Doppler shift. The most familiar example is the rise and then fall in pitch of the sound waves from an ambulance siren as it overtakes you, but light waves also show this effect.

9.a) It is the orbiting exoplanet pulling the star towards and away from our telescope, and so driving the sine wave of the star's radial velocity. The amplitude of this sine wave is dependent on the gravitational force with which the planet tugs its star, and therefore the mass of the planet. Specifically, the amplitude of the radial velocity sine wave is directly proportional to the ratio of the planet's mass and the star's mass. You could ask your more capable classes to explain why it is that a planet in a circular orbit produces a sine wave in the star's radial velocity - using the principles of simple harmonic motion. If the planet is in a very elliptical orbit, what effect would that have on the radial velocity plot?

In fact, since we do not know exactly how the plane of the exoplanet system is orientated relative to our telescope, we cannot calculate the exact mass of a planet discovered by the radial velocity method. If the exoplanet system is orientated almost side-on from our point of view (like a transiting system) even a low-mass planet can cause a large radial velocity in its star, whereas if the system is angled almost face-on to us then even a massive planet would cause only a slight velocity towards and away from our telescope. So in fact, all you can calculate from the radial velocity technique is $m.\sin(i)$, where m is the mass of the planet and i is the angle of inclination of the planetary system. If however, we discover a planet by the radial velocity technique and are then also able to detect it transiting we know that we are viewing that planetary system side-on; $i=90°$, $\sin(i)=1$, and we are therefore able to determine the actual mass of the exoplanet.

9. b) The period of the sine wave, i.e. the time taken for the planet to return to the same point in its path around the star, is the orbital period of the planet (its year length) – around 4 days in this case.

10. Knowing the radius of a spherical planet you can calculate its volume, and dividing the mass by this gives you the average density of the world. This is crucial information in trying to work out if the planet is mostly gas like Uranus or Neptune, or is predominantly rocky like the terrestrial planets of Earth, Mercury, Venus and Mars. This overall make-up of the planet, along with its orbit and thus how much heat it receives from its sun, are crucial factors for determining whether the planet offers a habitable environment suitable for life.

To conclude this lesson, and consolidate the learning objectives, you could have a short open discussion with the students on:

- How can we deduce the orbital period (year) of an exoplanet?
- How can we deduce the orbital distance of the exoplanet from its star?
- What effects is this likely to have on the exoplanet?
- How can we deduce the radius of an exoplanet?
- How can we deduce the mass of an exoplanet?
- How can we deduce what sort of planet we have discovered?
- What are the limitations of the transit and the radial velocity methods?
- What limitations are there to the sorts of exoplanets we are able to detect?

Note: All of the plots used in this work sheet are genuine data of discovered exoplanets. Fig.2 shows exoplanet HD 189733b, adapted from from Sing *et al.* (2009), Astronomy and Astrophysics, 505, p.891-899 [http://dx.doi.org/10.1051/0004-6361/200912776]. Fig.5 shows the radial velocity data from the discovery of 51 Pegasi b, the first exoplanet to be discovered orbiting a main-sequence star. You can download a regularly-updated spreadsheet of all confirmed exoplanets from http://exoplanet.eu.