The biomechanical response of the human body within a laterally oscillating parametrically excited system

or

How to most creatively injure yourself bungee jumping

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Adventure sports such as abseiling and skydiving attract enough scare stories, and bungee jumping is no exception. For example, everyone's heard of the friend-of-a-friend that was killed jumping from a 60 m high bridge in Australia because she'd accidentally been tied to a rope *80 m* long. Or the newly-weds that tried a tandem jump on their honeymoon, but didn't hold on tightly enough on the way down and cracked face-first back together at the end of the rope. It is even said that your eyeballs can pop out of the sockets and dangle around on stalks, Jim-Carrey-in-the-Mask-style, from the decelerating force. Fortunately, most of these are urban myths, and survive only due to their pub-gossip potential rather than accuracy.

Bungee jumping is, however, undeniably responsible for a range of serious medical complaints, including musculoskeletal pain in the neck and back, headaches, dizziness and blurred vision[1]. Thankfully, most of these symptoms have no lasting effects, yet there are tales of much rarer and more severe afflictions. Imagine having your leg broken when the bungee rope first snaps taut, and then having to bounce up and down on it for several minutes.

Null Hypothesis, being the guardian of scientific accuracy that it is, here reviews the medical facts of bungee-induced injuries and reports on a new model of the physics involved in bungee jumping. This model suggests an hitherto unanticipated method for harming yourself. It is even possible, under the right conditions, for bungee jumpers to cheat gravity itself.

Medical Mishaps

Some of the first medical research into bungee jumping found, perhaps unsurprisingly, that heart rate, blood pressure, and cortisol levels – all indicators of psychological stress – increase markedly in novice jumpers prior to a bungee[2]. These all decline rapidly again after the jump, and are replaced with a temporary increase in beta-endorphin – the hormone associated with feelings of euphoria.

More shocking is the great range of horrific injuries reported in the medical literature.

Below is our non-fatal Top 5, in some vague order of increasing nastiness, of when Bungee Jumps Go Bad.

5 Judging the bungee jump so that your head just dips into the water below may be exhilarating, but it's also an insanely good way of causing yourself some serious mess. If the height or your weight is even slightly miscalculated, then hitting water at speed is not much different from tarmac. This is exactly what happened to one San Franciscan jumper, who smacked face-first into the water, and continued down until waist-deep. He claimed to have experienced little pain at the time, but woke the next morning with his mug so bruised and swollen he could not open either eye. A CT scan revealed hair-line fractures in both his nose bone and the floor of his eye socket. [3]

4 Another consequence of head-dipping is the sudden pressure change as you plunge underwater. In one patient, who rather unfortunately had their head angled back at the time, this trapped a pocket of air in their nose. The shockwave of compressed gas pulsed through his sinuses and forced its way into the eye socket. [4]

3 'Reverse bungees', where the victim is catapulted upwards using a bungee with a high attachment point, have been increasing in popularity over the last few years. With this has come a disconcerting shift in the variety of bungee injuries, as the acceleration on an unprepared novice can approach that of a fighter pilot. In one case, this initial crushing force has been blamed for the sudden change in chest pressure that resulted in a collapsed lung. [5]

2 Number 2 in our list also relates to a reverse bungee. This time the rapid skyward acceleration forced a collection of clotted blood under the membrane covering the brain of the hapless adrenaline junkie.[6]

1 The all-time nastiest non-fatal bungee jumping incident resulted from a 'normal' jump. The sudden whip-like straining of the bungee cord cracked the man's spine and caused a 'unilateral locked facet' - dislocation of the vertebra. The spinal cord was damaged and the man left paralysed from the neck down. [7]

The Physics of Bungee

The vast majority of jumping injuries arise from the immense accelerations generated by reverse bungees, the pressure changes during head-dipping, or the sudden body-whip as the bungee cord first tightens. This third effect is the cause of most minor medical complaints. Bungee jumpers often fall in a characteristic horizontal body position, with arms swung back to the sides and rope trailing up from the attachment point by the feet. When the rope first pulls straight it starts to stretch, the strain force tugging back on the ankles. The body's center of mass (somewhere near the stomach) is forced round until it is directly beneath the attachment point. The head, the furthest point from this pivot, is viciously whipped around and puts an enormous strain on the bones and muscles of the back and neck. This whiplash is unlikely to ever be forceful enough to dislodge your eyeballs, although concerned jumpers can take certain steps to minimize the risk (see the box at the end of the article).

A computer model can be used to simulate the physics of a bungee jump to reveal the range of possible behaviours. The MADYMO software is one such simulation code, which treats a human as a jointed rigid body and the bungee cord as a light extensible rope, as shown in Figure 1.

The first level of analysis would be to treat the jumper as a simple mass on a spring. This situation represents the case of a dampened oscillator. The bungee jumper falls directly down before the rope is pulled straight, the strain forces decelerate the jumper until he is momentarily stationary, and then bounces him back up again. This repeats several times, with the rope gradually losing energy and the bounce amplitude decreasing.

The next stage is to model the stretching of the bungee cord more realistically, and then to allow the jumper to swing around on the end, away from the vertical axis. Such a model uses a series of linked formulas describing the changing extension of the cord, the angle of the cord to the vertical, the period of oscillation of the bouncing jumper, and so on. Input variables, such as the original length of the rope, the mass of the jumper, and the 'stretchiness' of the bungee rope can all be changed to study how the system behaves under different parameter values.

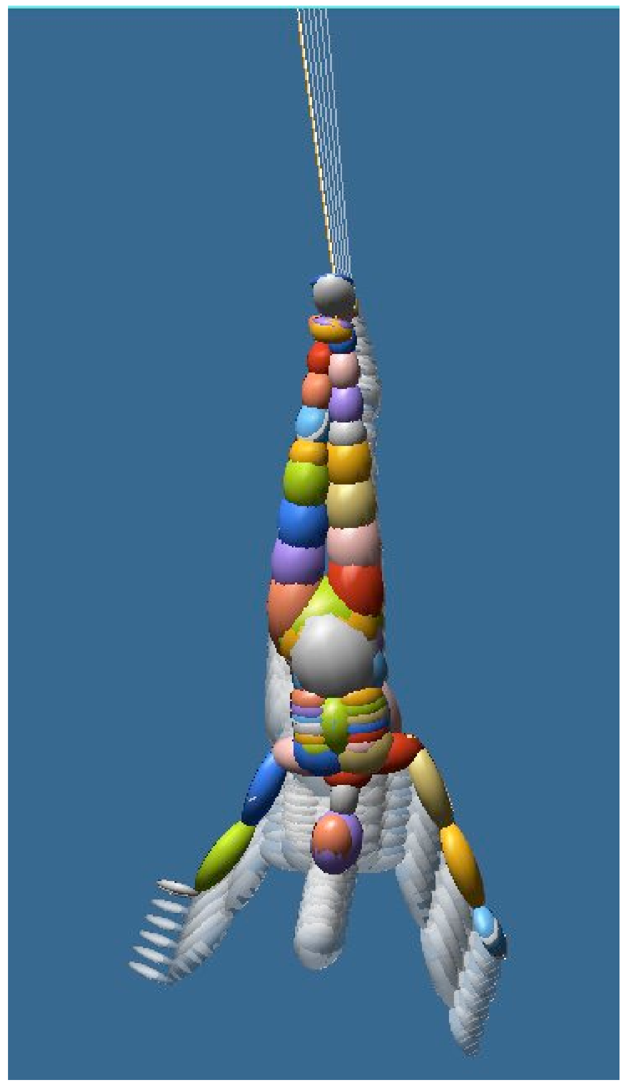

Figure 1. A MADYMO model of a bungee jumper being accelerated back upwards during a bounce.

The resultant equation created by combining such formulas is very similar in form to those used to describe other systems, such as the sway of buildings during earthquakes or the sickening roll of a ship in stormy seas. So, for example, the driving frequency would be the shaking ground, with the natural frequency of the system being whatever rate the building normally sways at, like a ruler held over the edge of a desk and twanged. In the specific case of a bungee jump, the driving frequency is how quickly the person bounces at the end of the rope.

Certain combinations of parameter values spell doom for the system. The equation becomes unstable and the solutions are unbounded - enormous oscillations are produced. The largest unstable region results from a '2:1 resonance'. This means that if the driving frequency is twice that of the natural frequency of the system then the whole thing catastrophically crashes. In the cases of the examples above this would signify the collapse of the building or the ship being forced to roll so much that it capsizes.

The implication of these mathematical instabilities for bungee jumping is that the up-and-down bouncing motion of the jumper can be converted into a side-to-side swinging movement. Certain combinations of the jumpers weight and rope length satisfy the criteria, with the MADYMO-modeled effects shown in Figure 2. For simplicity, the jumper is shown here as a point on the end of the rope, with position changing over time as the rope bounces and the lateral swings increase in amplitude. Whilst some might argue that this additional degree of freedom in the movement of a jumper would only add to the thrill and enjoyment of a bungee jump experience, the authors are rather alarmed by the popularity of jumping with long ropes from bridges spanning narrow canyons. The effects of a system resonance converting vertical bouncing to wide lateral swings could be somewhat unfortunate.

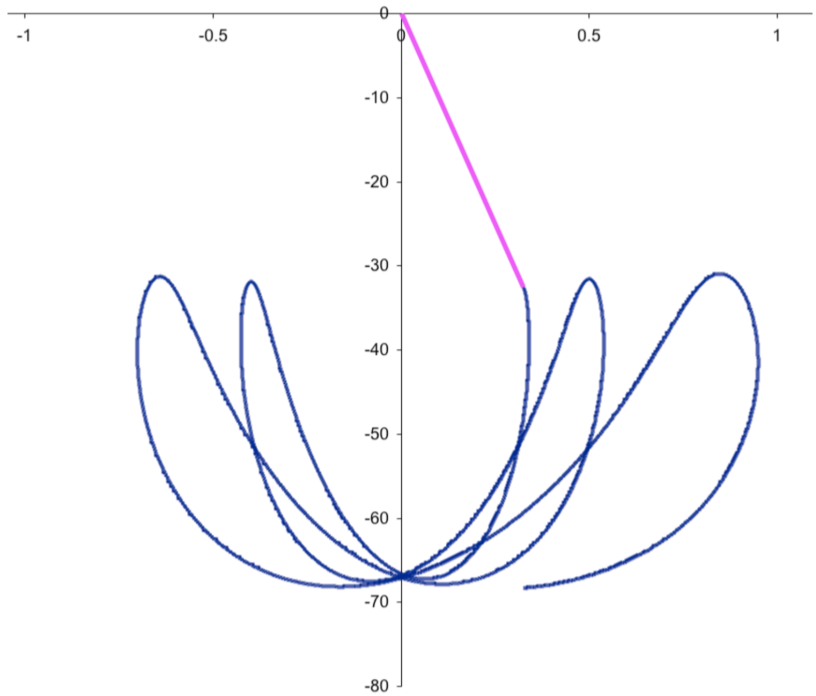

Figure 2. The side-to-side swinging of a jumper on the end of a bungee rope. These lateral oscillations increase in amplitude over time. The initial position of the rope is shown as the thick grey line.

A further complication in the modeling of the physics of bungee jumping is that the rope should not really be treated as a massless spring. They are bloody heavy. In fact, for the higher drops the weight of the rope itself can equal, if not exceed, that of the jumper. This similar mass condition results in a very interesting phenomenon. The bungee jumper accelerates faster than gravity.

In this more complex scenario the jumper is not strictly in free fall, but is attached to a heavy rope that is following him down. The slack rope snakes about as it drops, with different sections of it momentarily ceasing their descent. This can result in momentum transfer from the rope to the falling person, thus accelerating them downwards at a rate faster than g. A bungee jumper with a heavy rope for a short time falls to the ground faster than a skydiver.

So, the take home message? Bungee jumping, although a sport safely enjoyed by many can cause some horrific injuries. The risk of collapsed lungs and broken eye sockets is by now well documented, but this computer model has raised another interesting possibility for bodily damage. However, understanding the physics of 2:1 resonance is unlikely to be much consolation as you rebound crazily between the walls of a narrow canyon on a long bungee rope.

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The *Null Hypothesis* Guide to keeping your eyeballs where they belong.

The whipping-action is due to the fact that your centre of mass is not in line with the decelerating force from the stretching bungee rope and so rapidly pivots around your ankles. The head, being furthest from the point of rotation, experiences an enormous angular acceleration. In order to minimise this dangerous acceleration the head ought to be as close to the pivot point as possible. Therefore bungee jumping with the rope tied around the neck should be recommended on safety grounds (although the authors have not yet fully investigated other problems that may occur with this attachment).

Alternatively, jump head-first like you would off a swimming pool diving board, rather than the more heroic-looking swallow dive. This means that by the time you have fallen the length of the rope and it begins decelerating you your head is already directly beneath the point of rotation and your body attitude is changed only slightly. The resultant force is now almost completely downwards, pushing your viscera into your lungs and your brain into the top of your skull, but minimising the sideways whipping-action.